Powers and Roots

A power is also termed an exponent or index in mathematics and it is denoted by a superscript numeral.

The general definition of a power is that

$$x^n = \underbrace{x \times x \times x \ldots . \times x}_{n \text{ times}}.$$

The n^{th} root of a number y is a number r such that $r^{n} = y$.

Square and square root

The most simple power is the *square* which is equivalent to a power of 2. The square of a number is the result of multiplying it by itself.

If $r^2 = y$ then r can be said to be the square root of y. However if $r^2 = y$ then $(-r)^2 = y$, giving another square root. The non negative square root (or principal square root) is often what is meant by the square root. Also, the way the square root is often strictly used in mathematics (such as the formula solution of quadratic equations) implies that the non negative square root is a unique square root. To avoid ambiguity it is best to regard the square root as the principal square root only.

For example $3^2 = 3 \times 3 = 9$ and $\sqrt{9} = 3$.

Cube

The cub*e* is equivalent to a power of 3.

For example $5^3 = 5 \times 5 \times 5 = 125$ and $\sqrt[3]{125} = 5$.

Another example

$$6^5 = 6 \times 6 \times 6 \times 6 \times 6 = 7776.$$

Operations on Powers

The following identities provide means for simplifying powers.

1.

$$(xy)^p \equiv x^p y^p$$

For example (a) $6^2 = 2^2 3^2 = 4 \times 9 = 36$,

(b) $\sqrt{36} = \sqrt{4}\sqrt{9} = 2 \times 3 = 6$.

2.

$$x^p x^q = x^{(p+q)}$$

For example $2^2 2^3 = 2^{(2+3)} = 2^5 = 32$

3.

$$(x^p)^q = x^{pq}$$

For example $(2^3)^2 = 2^{(3\times 2)} = 2^6 = 64$.

Negative Power

A power of -1 represents the reciprocal of a number:

$$x^{-1} = \frac{1}{x}.$$

For example

$$2^{-1} = \frac{1}{2} = 0.5 .$$

For a general integer power:

$$x^{-n} = \frac{1}{x^n} \quad .$$

For example

$$5^{-3} = \frac{1}{5^3} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125} = 0.008\,.$$

Fractional Powers

A power of $\frac{1}{2}$ is interpreted as a square root.

For example $9^{\frac{1}{2}} = \sqrt{9} = 3$.

A power of $\frac{1}{3}$ is interpreted as a cube root.

For example $8^{\frac{1}{3}} = \sqrt[3]{8} = 2.$

The outcome of this is that roots are powers.

A power of $\frac{3}{4}$ is interpreted as

For example $16^{\frac{3}{4}} = (\sqrt[4]{16})^3 = \sqrt[4]{16^3} = 8$.

For example $4^{2\frac{1}{2}} = 4^2 \times 4^{\frac{1}{2}} = 16 \times 2 = 32$.

Negative Fractional Powers

For example $9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{3}$.

For example $4^{-2\frac{1}{2}} = \frac{1}{4^{2\frac{1}{2}}} = \frac{1}{32} = 0.03125$.

Zero Power

The result of raising a number to a power of zero is to give the result 1.

For example $2^0 = 1, (0.1)^0 = 1$.

Further worksheets on powers and roots can be found at Mathcentre[1].

[1]Mathcentre : Indices or Powers : Surds and other Roots